EFFECTIVE LAGRANGIAN AND DYNAMICAL SYMMETRY BREAKING IN $SU(2) \otimes U(1)$ NJL MODEL

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Abstract

Dynamical symmetry breaking and the formation of scalar condensates in the $SU(2) \otimes U(1)$ Nambu-Jona-Lasinio model with two coupling constants has been studied in the framework of the mean-field approximation. The bosonization procedures of the model are performed using the functional integration method. The possibility of the spontaneous CP symmetry breaking in the model under consideration has been shown. The mass spectrum of the bound states of fermions, as well as the effective Lagrangian of interacting scalar and pseudoscalar mesons are obtained.

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One of the modern problems is the investigation of nonperturbative effects in quantum chromodynamics (QCD) and obtaining the effective hadron Lagrangians. This area of low energy is difficult to study in the framework of QCD because coupling constant α_s is not a small parameter. Therefore, in this domain, some phenomenological models are used. Among them are a model based on local effective chiral Lagrangians (ECL) [1], [2], [3], a model of instanton vacuum [4], [5] (see also [6]), the Nambu-Jona-Lasinio (NJL) models [7] which use a contact four-quark interactions, etc.. In this letter the possibility of the spontaneous CP symmetry breaking in the NJL model is shown. It should be noted that the electric dipole moment of particles violates CP invariance and can be explained by the θ-term of QCD. Although the effect of CP violation in strong interactions is small, the investigation of mechanisms of CP violation is important.

Let us consider a four-fermion model with scalar-scalar and pseudoscalar-pseudoscalar interactions and possessing the internal symmetry group $SU(2)\otimes U(1)$ and two coupling constants:

$$\mathcal{L}(x) = -\overline{\psi}^n(x)(\gamma_\mu\partial_\mu + m_0)\psi^n(x) + \frac{F}{2}\left[\overline{\psi}^n(x)\psi^n(x)\right]^2 - \frac{G}{2}\left[\overline{\psi}^n(x)\gamma_5\tau^a\psi^n(x)\right]^2, \tag{1}$$

where τ^a $(a = 1, 2, 3)$ are the Pauli matrices, $\partial_\mu = (\partial/\partial x_i, -i\partial/\partial x_0)$ (x_0 is the time), $m_0 = \mathrm{diag}(m_{01}, m_{02})$; m_{01}, m_{02} are the current masses of fermions (quarks), γ_μ are the Dirac matrices, $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$, ψ is the doublet of fermions (u, d quarks). Here summation over colour fermion degrees of freedom $n = 1, 2, ..., N_C$ has been performed. We also assume equal current masses of fermions, $m_{01} = m_{02} \equiv m_0$. Lagrangian (1) is not invariant under γ_5-chiral transformations, i.e. $U_A(1)$-symmetry is broken. Note that the violation of $U_A(1)$-symmetry allows one to solve the problem of the differences of masses of η and η' mesons in the framework of $SU(3)_f \bigotimes SU(3)_f$-model. We shell investigate dynamical symmetry breaking (DSB) and the mass formation using functional integration method [8].

Considering the generating functional for Green's functions

$$Z[\overline{\eta}, \eta] = N_0 \int D\overline{\psi} D\psi \exp\left\{ i \int d^4x \left[\mathcal{L}(x) + \overline{\psi}^n(x)\eta^n(x) + \overline{\eta}^n(x)\psi^n(x) \right] \right\}, \tag{2}$$

where $\overline{\eta}^n$, η^n are external sources, and redefining the constant N_0, one can represent Eq. (2) as follows:

$$\begin{aligned} Z[\overline{\eta}, \eta] = N \int D\overline{\psi} D\psi D\Phi_0 D\tilde{\Phi}_a \exp\Big\{ i \int d^4x \Big[-\overline{\psi}^n(x) \Big[\gamma_\mu\partial_\mu + m_0 - f_0\Phi_0(x) \\ -ig_0\tilde{\Phi}_a(x)\gamma_5\tau^a \Big] \psi^n(x) - \frac{\mu^2}{2}\tilde{\Phi}_a^2(x) - \frac{M^2}{2}\Phi_0^2(x) + \overline{\psi}^n(x)\eta^n(x) + \overline{\eta}^n(x)\psi^n(x) \Big] \Big\}. \end{aligned} \tag{3}$$

We define constants $F = f_0^2/M^2$, $G = g_0^2/\mu^2$, where f_0, g_0 are dimensionless coupling constants and constants M, μ are mass-dimensional. One can integrate the functional integral in Eq. (3) over the fermion fields $\overline{\psi}$, ψ and obtain

$$Z[\overline{\eta}, \eta] = N \int D\Phi_0 D\tilde{\Phi}_a \exp\Big\{ iS[\Phi] + i \int d^4x d^4y \overline{\eta}^n(x) S_f(x, y) \eta^n(y) \Big\}, \tag{4}$$

$$\begin{aligned} S[\Phi] = -\frac{1}{2} \int d^4x \left[M^2\Phi_0^2(x) + \mu^2\tilde{\Phi}_a^2(x) \right] \\ -i\mathrm{tr} \ln \left[1 + \widehat{G} \left(f_0\Phi_0(x) + ig_0\tilde{\Phi}_a(x)\gamma_5\tau^a \right) \right], \end{aligned} \tag{5}$$

where we introduce the effective action for bosonic collective fields $S[\Phi]$. The fields $\Phi_0(x)$, $\tilde{\Phi}_a(x)$ can be identified with the σ-meson and triplet of pseudoscalar π_a-mesons [7]. The Green functions for free fermions $\widehat{G}$, and for fermions in the external collective fields $S_f(x, y)$ are solutions of the equations

$$(\gamma_\mu\partial_\mu + m_0)\widehat{G}(x, y) = -\delta(x - y), \tag{6}$$

$$\left[\gamma_\mu\partial_\mu + m_0 - f_0\Phi_0(x) - ig_0\tilde{\Phi}_a(x)\gamma_5\tau^a\right] S_f(x,y) = \delta(x-y). \tag{7}$$

In four-fermion model under consideration, the symmetric vacuum is not stable. As a result, the physical vacuum is reconstructed, and the appearance of the condensates leads to DSB of the initial $SU(2)\bigotimes U(1)$-symmetry. These condensates are coordinate-independent fields Φ_0, $\tilde{\Phi}_a$ (the mean field approximation) which obey Eq. (7). Eq. (7) in the momentum space for condensates Φ_0, $\tilde{\Phi}_a$ becomes

$$(i\widehat{p} - A)S_f(p) = 1, \tag{8}$$

where $\widehat{p} = p_\mu\gamma_\mu$, $p_\mu = (\mathbf{p}, ip_0)$, $A = -m_0 + f_0\Phi_0 + ig_0\tilde{\Phi}_a\gamma_5\tau^a$. Using the gauge $\Phi_0 \neq 0$, $\tilde{\Phi}_3 \neq 0$, $\tilde{\Phi}_1 = \tilde{\Phi}_2 = 0$ in which the matrix A is diagonal, we obtain from Eq. (8) the Green function

$$S_f(p) = \mathrm{diag}\left(\frac{-i\widehat{p} + m_1 + ig_0\tilde{\Phi}_3\gamma_5}{p^2+m^2}, \frac{-i\widehat{p} + m_1 - ig_0\tilde{\Phi}_3\gamma_5}{p^2+m^2}\right), \tag{9}$$

where the masses of fermions are defined as follows

$$m_1 = m_0 - f_0\Phi_0, \qquad m^2 = m_1^2 + g_0^2\tilde{\Phi}_3^2. \tag{10}$$

If the current fermion masses $m_0 = 0$, the dynamical fermion masses $m \neq 0$. It should be noted that the components containing the term $ig_0\tilde{\Phi}_3\gamma_5$ in Eq. (9) violate CP parity.

To obtain the vacuum condensates Φ_0, $\tilde{\Phi}_3$ from Eq. (5), one finds the equations for the fields $\Phi_0(x)$, $\tilde{\Phi}_3(x)$:

$$\begin{aligned} \frac{\delta S[\Phi]}{\delta\Phi_0(x)} &= -M^2\Phi_0(x) + if_0\mathrm{tr}S_f(x,x) = 0, \\ \frac{\delta S[\Phi]}{\delta\tilde{\Phi}_a(x)} &= -\mu^2\tilde{\Phi}_a(x) - g_0\mathrm{tr}\left[S_f(x,x)\gamma_5\tau^a\right] = 0. \end{aligned} \tag{11}$$

With the help of Eqs. (9), (11), we obtain gap equations for the vacuum condensates:

$$\begin{aligned} M^2\Phi_0 &= 2f_0 I m_1, \\ \mu^2\tilde{\Phi}_3 &= -2g_0^2 I\tilde{\Phi}_3, \end{aligned} \tag{12}$$

where

$$I = \frac{iN_C}{4\pi^4}\int\frac{d^4p}{p^2+m^2} \qquad (d^4p = id^3p dp_0). \tag{13}$$

The quadratically divergent integral in Eq. (13) can be evaluated with help of the momentum cutoff Λ, and gap equations (12) possess non-trivial ($\Phi_0 \neq 0$, $\tilde{\Phi}_3 \neq 0$) non-analytic in constants F, G solutions [7] at $N_C F\Lambda > 2\pi^2$, $N_C G\Lambda > 2\pi^2$. Thus, even at zero current masses of fermions ($m_0 = 0$), as a result of the phase transition, they acquire non-zero masses, and the massive states of fermions correspond to the minimum of effective potential [9], [10]. The appearance of the non-zero vacuum condensate $\tilde{\Phi}_3 \neq 0$ indicates the spontaneous CP symmetry breaking.

From Eqs. (12) one arrives at the relationship

$$F m_0 = (F - G) f_0 \Phi_0. \tag{14}$$

It follows from Eq. (14) that if the bare masses of fermions $m_0 = 0$ and $\Phi_0 \neq 0$, the equality $F = G$ is valid, and we come to the model with only one coupling constant.

The parameter cutoff (Λ) specifies the region of non-local fermion-antifermion (quark-antiquark) interactions and this region is determined by the size $1/\Lambda$.

Expending the logarithm in Eq. (5) in small fluctuations of fields $\Phi'_0(x)$, $\tilde{\Phi}'_a(x)$ one obtains:

$$\Phi_0(x) = \Phi_0 + \Phi'_0(x), \qquad \tilde{\Phi}_3(x) = \tilde{\Phi}_3 + \tilde{\Phi}'_3(x),$$

where condensates Φ_0, $\tilde{\Phi}_3$ are the solutions of gap equations (12). Then the effective action (5) becomes

$$\begin{gathered} S[\Phi'] = -\frac{1}{2}\int d^4x d^4y \Phi'_A(x)\Delta^{-1}_{AB}(x,y)\Phi'_B(y) + \sum_{n=3}^{\infty}\frac{i}{n}\mathrm{tr}\left[S_f\left(f_0\Phi'_0 + ig_0\tilde{\Phi}'_a\gamma_5\tau^a\right)\right]^n, \\ \Delta^{-1}_{AB}(p) = -ig_A g_B \mathrm{tr}\left[\int\frac{d^4k}{(2\pi^4)}S_f(k)T_A S_f(k-p)T_B\right] + \delta_{AB}M_A^2, \end{gathered} \tag{15}$$

where $g_A = (f_0, g_0)$, $M_A = (M, \mu)$, $T_A = (1, i\gamma_5\tau^a)$, $\Phi_A = (\Phi_0, \tilde{\Phi}_a)$. Using the quark propagator (9), gap equations (12), and calculating the nonzero elements $\Delta^{-1}_{AB}(p)$ in Eq. (15), we find

$$\begin{aligned} \Delta^{-1}_{11}(p) = \Delta^{-1}_{22}(p) &= p^2 Z_3^{-1} + \mathcal{O}(g_0^2), \\ \Delta^{-1}_{33}(p) &= \left(p^2 + 4g_0^2\tilde{\Phi}_3^2\right) Z_3^{-1} + \mathcal{O}(g_0^2), \\ \Delta^{-1}_{00}(p) &= \frac{2f_0^2 m_0 I}{m_0 - m_1} + \left(p^2 + 4m_1^2\right)\frac{f_0^2}{g_0^2}Z_3^{-1} + \mathcal{O}(f_0^2), \end{aligned} \tag{16}$$

$$\Delta_{03}^{-1}(p) = -4m_1 f_0 \tilde{\Phi}_3 Z_3^{-1} + \mathcal{O}(f_0 g_0),$$

where the constant of renormalization is given by

$$Z_3^{-1} = -\frac{i g_0^2 N_C}{4\pi^4} \int \frac{d^4 q}{(q^2+m^2)^2}. \tag{17}$$

Now we introduce the renormalized fields $\tilde{\Phi}_a(x) = Z_3^{-1/2} \tilde{\Phi}'_a(x)$, $\Phi_0(x) = (f_0/g_0) Z_3^{-1/2} \Phi'_0(x)$ and coupling constants $g^2 = Z_3 g_0^2$, $f^2 = Z_3 f_0^2$. Logarithmic and quadratic integrals are connected by the relation [11]

$$Z_3^{-1} = \frac{g_0^2}{m^2} I - \frac{g_0^2}{4\pi^2}. \tag{18}$$

It follows from Eq. (17) that the expansion in $g^2/4\pi^2$, $f^2/4\pi^2$ corresponds to the $1/N_C$ expansion. Neglecting the terms (radiation corrections) $\mathcal{O}(g^2)$, $\mathcal{O}(f^2)$, $\mathcal{O}(fg)$, one finds the renormalized, quadratic in the boson fields, effective Lagrangian

$$\mathcal{L}_{free} = -\frac{1}{2} \left[(\partial_\mu \Phi_A(x))^2 + m_{AB}^2 \Phi_A(x) \Phi_B(x) \right], \tag{19}$$

where $\Phi_A = (\Phi_0, \tilde{\Phi}_a)$ and the elements of the mass matrix read

$$\begin{gathered} m_{00}^2 = 4m_1^2 + \frac{2m_0 m^2}{m_0 - m_1}, \qquad m_{11}^2 = m_{22}^2 = 0, \\ m_{03}^2 = -4m_1 g \tilde{\Phi}_3, \qquad m_{33}^2 = 4g^2 \tilde{\Phi}_3^2. \end{gathered} \tag{20}$$

The mass spectrum of the bosonic fields $\Phi_A(x)$ can be obtained by diagonalizing the mass matrix m_{AB} in Eq. (19). In accordance with Eq. (20), the masses of the fields $\tilde{\Phi}_1(x)$, $\tilde{\Phi}_2(x)$ are zero, in agreement with the Goldstone theorem [12]. The fields $\Phi_0(x)$, $\tilde{\Phi}_3(x)$ possess nonzero masses. One may make the $SO(2)$-transformations

$$\Phi'_0(x) = \Phi_0(x)\cos\alpha - \tilde{\Phi}_3(x)\sin\alpha, \quad \tilde{\Phi}'_3(x) = \Phi_0(x)\sin\alpha + \tilde{\Phi}_3(x)\cos\alpha, \tag{21}$$

where $\tan 2\alpha = 2m_{00}^2/(m_{33}^2 - m_{00}^2)$ to diagonalize the mass matrix m_{AB}. As a result, we obtain the masses of bosonic fields $\Phi'_0(x)$, $\tilde{\Phi}'_3(x)$:

$$m_{00}'^2 = m_{00}^2 \cos^2\alpha + m_{33}^2 \sin^2\alpha - m_{03}^2 \sin 2\alpha, \tag{22}$$

$$m'^2_{33} = m^2_{00}\sin^2\alpha + m^2_{33}\cos^2\alpha + m^2_{03}\sin 2\alpha,$$

but the fields $\tilde{\Phi}_1(x)$, $\tilde{\Phi}_2(x)$ remain massless. According to Eq. (22) the field $\tilde{\Phi}_3(x)$ acquires small mass due to the CP-violating condensate $\tilde{\Phi}_3$. The transformations of the collective fields (21) are induced by the corresponding transformations of fermionic fields $\psi(x)$.

Now we evaluate the effective Lagrangian of interacting bosonic fields by counting the components in the sum (15) with $n = 3$ and $n = 4$ in the case of small parameters of expansion $g^2/4\pi^2 < 1$, $f^2/4\pi^2 < 1$. Note that the fermion loops at $n > 4$ give small convergent corrections. We make calculations of three- and four-point functions at the particular case of equal coupling constants $F = G$ $(f = g)$ and skipping the small CP-violating condensate $\tilde{\Phi}_3 = 0$. After renormalization, we obtain the effective Lagrangian of interacting bosonic fields to an accuracy $\mathcal{O}(g^2)$, as follows (see also [6]):

$$\begin{aligned} \mathcal{L}_{int}(x) &= 2gm\left(\Phi_0^3(x) + \Phi_0(x)\tilde{\Phi}_a^2(x)\right) \\ &-\frac{g^2}{2}\left[\Phi_0^4(x) + \left(\tilde{\Phi}_a^2(x)\right)^2 + 6\Phi_0^2(x)\tilde{\Phi}_a^2(x)\right]. \end{aligned} \tag{23}$$

We have just considered the NJL model with $SU(2)\bigotimes U(1)$ internal symmetry group and two coupling constants which can provide the dynamical CP symmetry violation. The scalar collective field Φ_0 is associated with σ-meson, and the pseudoscalar bosons $\tilde{\Phi}_a$ are associated with the triplet of pions. As usual [7], pions play the role of the Goldstone bosons. The four-fermion interaction is some approximation to the real quark interactions. However, to take into consideration confinement of quarks, one has to modify the model.

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